

Quark/lepton mass and mixing in S_3 invariant model and CP-violation of neutrino

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Abstract

Weak bases of flavors (u, c) , (d, s) , (e, μ) , (ν_e, ν_μ) are assumed as the S_3 doublet and t , b , τ , ν_τ are the S_3 singlet and further there are assumed S_3 doublet Higgs (H_1, H_2) and S_3 singlet Higgs H_S . We suggest an S_3 invariant model in which the Yukawa interactions constructed from these S_3 doublets and singlets are S_3 invariant. In this model, we can explain the quark sector mass hierarchy, quark mixing V_{CKM} and measure of CP violation naturally. In the leptonic sector, neutrino masses are assumed to be constructed through the see-saw mechanism from the Majorana mass. The tri-bimaximal-like character of neutrino mixing V_{MNS} can be explained dynamically without any other symmetry restrictions. It is predicted that a quasi-degenerate mass spectroscopy of neutrino is favorable, and values of $|V_{\text{MNS}}|_{13}$, CP violation invariant measure J and the effective Majorana mass $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ are not so tiny.

Keywords : quark/lepton mass, quark/neutrino mixing, S_3 symmetry, CP violation of neutrino

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I. INTRODUCTION

In present elementary particle physics, the problem of the origin of quark/lepton mass and mixing is the most interesting and challenging subject, for the exploring this problem leads to the finding of a clue of a new theory over the standard theory of the elementary particle physics. Although the quark mass hierarchy and quark mixing V_{CKM} have been explained successfully by many authors [1–9], but neutrino mass hierarchy and mixing V_{MNS} having the large mixing character [10–12] is not explained sufficiently. Especially, smallness of neutrino mass and tri-bimaximal-like mixing nature of neutrino are not explained satisfactorily in the same footing as discussion of quark mass and mixing.

In these circumstances, many models in which flavors of quark and lepton are governed by the symmetry group S_3 [13–22], S_4 or A_4 [23–25] have been analyzed. Fundamental scenario to these models is that quark and lepton flavors and further Higgs fields are considered as to be governed by the discrete symmetry, S_3 , S_4 or A_4 group, and physical neutrinos are Majorana neutrino induced from see-saw mechanism through the mixing with right handed Majorana neutrino. Furthermore, almost models except ours [19] have considered the additional constraint, Z_2 , S_2 symmetry or $\mu - \tau$ symmetry, for explaining the tri-bimaximal-like mixing nature of neutrino. In contrast, we explained this tri-bimaximal-like mixing nature by a dynamical mechanism, in which neutrino mass matrix is induced from the mixing between Dirac mass and Majorana mass of neutrino, and the hierarchy between masses of S_3 singlet and doublet.

In our previous paper [19], we used a standard Yukawa interactions modified in order to make mass matrices for quark and lepton Hermit. But this modification of standard Yukawa interaction violates the conservation of hyper charge Y , then in the present work, we use a standard Yukawa interaction without any modification. Further, CP phases, one Dirac and two Majorana phases in neutrino mixing, are analyzed and CP-violation measure J and effective Majorana mass $| \langle m \rangle |$ in neutrino-less double β decay $((\beta\beta)_{0\nu})$ are estimated.

II. S_3 INVARIANT MODEL

First, we explain our S_3 invariant model. We assume that S_3 symmetry governs the generations of quark and lepton (charged lepton and Dirac neutrino). Weak bases of flavors

$(u, c)_{L,R}, (d, s)_{L,R}, (e, \mu)_{L,R}, (\nu_e, \nu_\mu)_{L,R}$ are assumed as the S_3 doublets and $t_{L,R}, b_{L,R}, \tau_{L,R}, \nu_{\tau L,R}$ are the S_3 singlet. Further there are assumed S_3 doublet Higgs (H_1, H_2) and S_3 singlet Higgs H_S . If we represent the fields of quark and lepton as f , our S_3 model are composed of the following fields,

$$\begin{aligned} S_3 \text{ singlet} &: f_S^{L,R}, H_S, \\ S_3 \text{ doublet} &: \mathbf{f}_D^{L,R} = (f_1^{L,R}, f_2^{L,R})^T, \mathbf{H}_D = (H_1, H_2)^T. \end{aligned} \quad (1)$$

and the fields $f^{L,R}$ represent the following quarks and leptons,

$$\begin{pmatrix} f_1^{L,R} \\ f_2^{L,R} \\ f_S^{L,R} \end{pmatrix} = \begin{pmatrix} u_{L,R} \\ c_{L,R} \\ t_{L,R} \end{pmatrix}, \begin{pmatrix} d_{L,R} \\ s_{L,R} \\ b_{L,R} \end{pmatrix}, \begin{pmatrix} \nu_{eL,R} \\ \nu_{\mu L,R} \\ \nu_{\tau L,R} \end{pmatrix}, \begin{pmatrix} e_{L,R} \\ \mu_{L,R} \\ \tau_{L,R} \end{pmatrix}. \quad (2)$$

In the $SU(2)_L$ gauge space, Higgs fields \mathbf{H}_D, H_S are $SU(2)_L$ doublets; $\mathbf{H}_D = (\mathbf{H}_D^+, \mathbf{H}_D^0)^T$, $H_S = (H_S^+, H_S^0)^T$, respectively. For quark fields, $Q_1^L = (u_L, d_L)^T$, $Q_2^L = (c_L, s_L)^T$, $Q_S^L = (t_L, b_L)^T$ are $SU(2)_L$ doublets, and $d_1^R = d_R, d_2^R = s_R, d_S^R = b_R, u_1^R = u_R, u_2^R = c_R, u_S^R = t_R$ are $SU(2)_L$ singlets, and for leptons, $L_1^L = (\nu_{eL}, e_L)^T$, $L_2^L = (\nu_{\mu L}, \mu_L)^T$, $L_S^L = (\nu_{\tau L}, \tau_L)^T$ are $SU(2)_L$ doublets, and $l_1^R = e_R, l_2^R = \mu_R, l_S^R = \tau_R, \nu_1^R = \nu_{eR}, \nu_2^R = \nu_{\mu R}, \nu_S^R = \nu_{\tau R}$ are $SU(2)_L$ singlets.

We start from the standard Yukawa interaction,

$$-\mathcal{L}_D^f = \sum_{i,j,k=1,2,S} [\Gamma_{ijk}^d \overline{Q}_i^L H_j d_k^R + \Gamma_{ijk}^u \overline{Q}_i^L \epsilon H_j^* u_k^R + \Gamma_{ijk}^l \overline{L}_i^L H_j l_k^R + \Gamma_{ijk}^\nu \overline{L}_i^L \epsilon H_j^* \nu_k^R] + h.c., \quad (3)$$

where Γ_{ijk}^f are complex interaction strengths and ϵ is the 2×2 antisymmetric tensor. We assume the following S_3 invariant mass Lagrangian for quarks and leptons under the spontaneous symmetry breaking of vacuum $\langle H_S \rangle = (0, H_S^0)^T$, $\langle \mathbf{H}_D \rangle = (0, \mathbf{H}_D^0)^T$,

$$\begin{aligned} -\mathcal{L}_D^{d,l} &= \Gamma_S^{d,l} \overline{f}_S^L f_S^R H_S^0 + \Gamma_{D1}^{d,l} \overline{\mathbf{f}}_D^L \mathbf{f}_D^R H_S^0 + \Gamma_{D2}^{d,l} [(\overline{f}_1^L f_2^R + \overline{f}_2^L f_1^R) H_1^0 + (\overline{f}_1^L f_1^R - \overline{f}_2^L f_2^R) H_2^0] \\ &\quad + \Gamma_{D3}^{d,l} (\overline{\mathbf{f}}_D^L \mathbf{H}_D^0 f_S^R + \overline{f}_S^L \mathbf{H}_D^{0*T} \mathbf{f}_D^R) + h.c., \end{aligned} \quad (4)$$

for down-type quark and charged lepton

$$\begin{aligned} -\mathcal{L}_D^{u,\nu} &= \Gamma_S^{u,\nu} \overline{f}_S^L f_S^R H_S^{0*} + \Gamma_{D1}^{u,\nu} \overline{\mathbf{f}}_D^L \mathbf{f}_D^R H_S^{0*} + \Gamma_{D2}^{u,\nu} [(\overline{f}_1^L f_2^R + \overline{f}_2^L f_1^R) H_1^{0*} + (\overline{f}_1^L f_1^R - \overline{f}_2^L f_2^R) H_2^{0*}] \\ &\quad + \Gamma_{D3}^{u,\nu} (\overline{\mathbf{f}}_D^L \mathbf{H}_D^{0*} f_S^R + \overline{f}_S^L \mathbf{H}_D^{0*T} \mathbf{f}_D^R) + h.c., \end{aligned}$$

for up-type quark and Dirac neutrino

where we used the fact that the S_3 doublet can be made from the tensor product of $\overline{\mathbf{f}}_D^L$ and \mathbf{f}_D^R [13–15] as

$$\begin{pmatrix} \overline{f}_1^L f_2^R + \overline{f}_2^L f_1^R \\ \overline{f}_1^L f_1^R - \overline{f}_2^L f_2^R \end{pmatrix}.$$

These mass Lagrangian (4) are almost similar to ones proposed in our previous paper [19], in which H_1^0 coupled to $\overline{f}_2^L f_1^R$ and $\overline{f}_S^L f_1^R$ in $\mathcal{L}_D^{d,l}$ and H_1^{0*} coupled to $\overline{f}_2^L f_1^R$ and $\overline{f}_S^L f_1^R$ in $\mathcal{L}_D^{u,\nu}$ are interchanged. In our previous model, the mass matrices produced from the S_3 invariant mass Lagrangian are Hermit, but the Yukawa interaction for these mass Lagrangian dose not conserve the hyper charge Y .

From the $SU(2)_L$ gauge freedom of fields H_S^0 , $f_S^{L,R}$, $f_1^{L,R}$ and $f_2^{L,R}$, we can choose the phase of $f_S^{L,R}$ as

$$\begin{aligned} \text{phase of } H_S^0 - \text{phase of } f_i^L + \text{phase of } f_i^R &= 0, \\ i &= S, 1, 2 \\ \text{phase of } f_S^R &= \text{phase of } f_1^R = \text{phase of } f_2^R. \end{aligned} \quad (5)$$

Then we can get the mass matrices $M_{d,l}$ for down-type quark and charged lepton, and $M_{u,\nu}$ for up-type quark and neutrino Dirac mass as follows;

$$\begin{aligned} -\mathcal{L}_D^f &= \overline{f}^L M_f f^R + h.c., \quad f = d, u, l, \nu, \\ M_{d,l} &= \begin{pmatrix} \mu_1^{d,l} + \mu_2^{d,l} e^{i\phi_2} & \lambda \mu_2^{d,l} e^{i\phi_1} & \lambda \mu_3^{d,l} e^{i\phi_1} \\ \lambda \mu_2^{d,l} e^{i\phi_1} & \mu_1^{d,l} - \mu_2^{d,l} e^{i\phi_2} & \mu_3^{d,l} e^{i\phi_2} \\ \lambda \mu_3^{d,l} e^{i\phi_1} & \mu_3^{d,l} e^{i\phi_2} & \mu_0^{d,l} \end{pmatrix}, \\ M_{u,\nu} &= \begin{pmatrix} \mu_1^{u,\nu} + \mu_2^{u,\nu} e^{-i\phi_2} & \lambda \mu_2^{u,\nu} e^{-i\phi_1} & \lambda \mu_3^{u,\nu} e^{-i\phi_1} \\ \lambda \mu_2^{u,\nu} e^{-i\phi_1} & \mu_1^{u,\nu} - \mu_2^{u,\nu} e^{-i\phi_2} & \mu_3^{u,\nu} e^{-i\phi_2} \\ \lambda \mu_3^{u,\nu} e^{-i\phi_1} & \mu_3^{u,\nu} e^{-i\phi_2} & \mu_0^{u,\nu} \end{pmatrix}, \end{aligned} \quad (6)$$

where we use the following parameterization,

$$\begin{aligned} \mu_0^f &= \langle \Gamma_S^f | H_S^0 \rangle_{SS}, \quad \mu_1^f = \langle \Gamma_{D1}^f | H_S^0 \rangle_{11,22}, \quad \mu_2^f = \langle \Gamma_{D2}^f | H_2^0 \rangle_{11,22}, \\ \lambda \mu_2^f &= \langle \Gamma_{D2}^f | H_1^0 \rangle_{12,21}, \quad \mu_3^f = \langle \Gamma_{D3}^f | H_2^0 \rangle_{2S,S2}, \quad \lambda \mu_3^f = \langle \Gamma_{D3}^f | H_1^0 \rangle_{1S,S1}, \\ \lambda &= \frac{|H_1^0|}{|H_2^0|}, \quad \phi_1 = \text{phase of } H_1^0 - \text{phase of } H_S^0, \quad \phi_2 = \text{phase of } H_2^0 - \text{phase of } H_S^0. \end{aligned}$$

Yukawa interaction strengths Γ_D for the interaction including \mathbf{f}_D or \mathbf{H}_D^0 are considered to be very small compared to Γ_S for the interaction including f_S^0 and H_S^0 , then $\mu_1, \mu_2, \mu_3 \ll$

μ_0 . Thus the mass hierarchy of d -type and u -type quarks and charged leptons are realized in our model.

For neutrino mass, we assume that there are very large Majorana masses constructed from the right-handed neutrinos, the existence of which are suggested in the $SO(10)$ GUT, and from this Majorana mass we can get the very small neutrino masses through the see-saw mechanism [26]. We assume that the Majorana mass is constructed as S_3 invariant containing only right handed neutrino $\nu_D^R = (\nu_1^R, \nu_2^R)^T$, ν_S^R and has no Higgs field [15],

$$\mathcal{L}_M = \frac{1}{2}\Gamma_S^M(\nu_S^R)^T C^{-1}\nu_S^R + \frac{1}{2}\Gamma_D^M(\nu_D^R)^T C^{-1}\nu_D^R + h.c., \quad (7)$$

where C is a charge conjugation matrix. This Majorana mass term is expressed as

$$\mathcal{L}_M = \frac{1}{2}(\nu_R)^T C^{-1} M_M \nu_R + h.c.,$$

$$M_M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_0 \end{pmatrix}, \quad (8)$$

using the next parametrization

$$M_0 = \langle \Gamma_S^M \rangle_{SS}, \quad M_1 = \langle \Gamma_D^M \rangle_{11,22}.$$

If $M_1 \ll M_0$, as the case of Dirac neutrino mass, we can explain the tri-bimaximal-like mixing character of neutrino without any other symmetry restriction. We will discuss this problem in section 4, in detail.

III. QUARK MASS AND MIXING

In this section, we consider the quark mass and mixing in detail. We assumed the Yukawa interaction describing the masses for d -type and u -type quark as Eq. (4) and the mass matrices for these Yukawa interactions were expressed in Eq. (6),

$$-\mathcal{L}_D^f = \overline{f^L} M_f f^R + h.c., \quad f = d, u, ,$$

$$f^{L,R} = \begin{pmatrix} d_{L,R} \\ s_{L,R} \\ b_{L,R} \end{pmatrix}, \quad M_d = \begin{pmatrix} \mu_1^d + \mu_2^d e^{i\phi_2} & \lambda \mu_2^d e^{i\phi_1} & \lambda \mu_3^d e^{i\phi_1} \\ \lambda \mu_2^d e^{i\phi_1} & \mu_1^d - \mu_2^d e^{i\phi_2} & \mu_3^d e^{i\phi_2} \\ \lambda \mu_3^d e^{i\phi_1} & \mu_3^d e^{i\phi_2} & \mu_0^d \end{pmatrix},$$

$$f^{L,R} = \begin{pmatrix} u_{L,R} \\ c_{L,R} \\ t_{L,R} \end{pmatrix}, \quad M_u = \begin{pmatrix} \mu_1^u + \mu_2^u e^{-i\phi_2} & \lambda \mu_2^u e^{-i\phi_1} & \lambda \mu_3^u e^{-i\phi_1} \\ \lambda \mu_2^u e^{-i\phi_1} & \mu_1^u - \mu_2^u e^{-i\phi_2} & \mu_3^u e^{-i\phi_2} \\ \lambda \mu_3^u e^{-i\phi_1} & \mu_3^u e^{-i\phi_2} & \mu_0^u \end{pmatrix}.$$

Because mass matrices M_d and M_u are complex symmetric ones, these matrices are diagonalized by the unitary matrix U and V as (see Appendix)

$$V_d^\dagger M_d U_d = \text{diag}[m_d, m_s, m_b], \quad V_d = U_d^* S_d^\dagger, \quad S_d = \text{diag}[e^{i\alpha_d}, e^{i\beta_d}, e^{i\gamma_d}],$$

$$\alpha_d, \beta_d, \gamma_d \text{ are arbitrary real constants,}$$

$$(d_R^m, s_R^m, b_R^m)^T = U_d^\dagger (d_R, s_R, b_R)^T, \quad (d_L^m, s_L^m, b_L^m)^T = V_d^\dagger (d_L, s_L, b_L)^T, \quad (9)$$

$$V_u^\dagger M_u U_u = \text{diag}[m_u, m_c, m_t], \quad V_u = U_u^* S_u^\dagger, \quad S_u = \text{diag}[e^{i\alpha_u}, e^{i\beta_u}, e^{i\gamma_u}],$$

$$\alpha_u, \beta_u, \gamma_u \text{ are arbitrary real constants,}$$

$$(u_R^m, c_R^m, t_R^m)^T = U_u^\dagger (u_R, c_R, t_R)^T, \quad (u_L^m, c_L^m, t_L^m)^T = V_u^\dagger (u_L, c_L, t_L)^T, \quad (10)$$

where (m_d^2, m_s^2, m_b^2) and (m_u^2, m_c^2, m_t^2) are eigenvalues of $M_d M_d^\dagger$ and $M_u M_u^\dagger$, respectively, and (d^m, s^m, b^m) and (u^m, c^m, t^m) are the mass eigen states for the weak basis (d, s, b) and (u, c, t) , respectively. As a result, the CKM mixing matrix V_{CKM} in the weak charged interaction is expressed as

$$V_{\text{CKM}} = V_u^\dagger V_d = S_u U_u^T U_d^* S_d^\dagger. \quad (11)$$

This V_{CKM} matrix can be parametrized by three mixing angles θ_{12} , θ_{23} , θ_{13} , and a CP-violating phase δ after adjusting 6 arbitrary phases in S_d , S_u , where 1 of 6 phases can be settled as 0. Standard expression of this matrix is written as

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (12)$$

where $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$, δ is CP-violating phase.

We calculate eigenvalues of quark masses and diagonalization matrices U analytically, under the assumption $\mu_1, \mu_2, \mu_3 \ll \mu_0$ and $\lambda = \frac{|H_1^0|}{|H_2^0|} \ll 1$.

◇ d -type quark:

$$m_d \approx \left[\mu_1^{d2} + (1 + \lambda^2) \mu_2^{d2} + 2\mu_2^d \sqrt{(\cos^2 \phi_2 + \lambda^2 \cos^2 \phi_1) \mu_1^{d2} + \lambda^2 \sin^2(\phi_1 - \phi_2) \mu_2^{d2}} \right]^{1/2},$$

$$m_s \approx \left[\mu_1^{d2} + (1 + \lambda^2) \mu_2^{d2} - 2\mu_2^d \sqrt{(\cos^2 \phi_2 + \lambda^2 \cos^2 \phi_1) \mu_1^{d2} + \lambda^2 \sin^2(\phi_1 - \phi_2) \mu_2^{d2}} \right]^{1/2}, \quad (13)$$

$$m_b \approx \mu_0^d,$$

$$U_d \approx \begin{pmatrix} \cos \theta_d & \sin \theta_d e^{i\theta_3^d} & \frac{\lambda \mu_3^d}{\mu_0^d} e^{-i\phi_1} \\ -\sin \theta_d e^{-i\theta_3^d} & \cos \theta_d & \frac{\mu_3^d}{\mu_0^d} e^{-i\phi_2} \\ \frac{\mu_3^d}{\mu_0^d} (-\lambda \cos \theta_d e^{i\phi_1} + \sin \theta_d e^{i\phi_2 - i\theta_3^d}) & -\frac{\mu_3^d}{\mu_0^d} (\lambda \sin \theta_d e^{i\phi_1 + i\theta_3^d} + \cos \theta_d e^{i\phi_2}) & 1 \end{pmatrix}. \quad (14)$$

$$\tan \theta_3^d = \frac{\mu_2^d \sin(\phi_1 - \phi_2)}{\mu_1^d \cos \phi_1},$$

$$\tan \theta_d = -\frac{\lambda \sqrt{\mu_1^{d2} \cos^2 \phi_1 + \mu_2^{d2} \sin^2(\phi_1 - \phi_2)}}{\sqrt{(\cos^2 \phi_2 + \lambda^2 \cos^2 \phi_1) \mu_1^{d2} + \lambda^2 \sin^2(\phi_1 - \phi_2) \mu_2^{d2}} + \mu_1^d \cos \phi_2}.$$

◇ u -type quark:

$$m_u \approx \left[\mu_1^{u2} + (1 + \lambda^2) \mu_2^{u2} + 2\mu_2^u \sqrt{(\cos^2 \phi_2 + \lambda^2 \cos^2 \phi_1) \mu_1^{u2} + \lambda^2 \sin^2(\phi_1 - \phi_2) \mu_2^{u2}} \right]^{1/2},$$

$$m_c \approx \left[\mu_1^{u2} + (1 + \lambda^2) \mu_2^{u2} - 2\mu_2^u \sqrt{(\cos^2 \phi_2 + \lambda^2 \cos^2 \phi_1) \mu_1^{u2} + \lambda^2 \sin^2(\phi_1 - \phi_2) \mu_2^{u2}} \right]^{1/2}, \quad (15)$$

$$m_t \approx \mu_0^u,$$

$$U_u \approx \begin{pmatrix} \cos \theta_u & \sin \theta_u e^{i\theta_3^u} & \frac{\lambda \mu_3^u}{\mu_0^u} e^{i\phi_1} \\ -\sin \theta_u e^{-i\theta_3^u} & \cos \theta_u & \frac{\mu_3^u}{\mu_0^u} e^{i\phi_2} \\ \frac{\mu_3^u}{\mu_0^u} (-\lambda \cos \theta_u e^{-i\phi_1} + \sin \theta_u e^{-i\phi_2 - i\theta_3^u}) & -\frac{\mu_3^u}{\mu_0^u} (\lambda \sin \theta_u e^{-i\phi_1 + i\theta_3^u} + \cos \theta_u e^{-i\phi_2}) & 1 \end{pmatrix}. \quad (16)$$

$$\tan \theta_3^u = -\frac{\mu_2^u \sin(\phi_1 - \phi_2)}{\mu_1^u \cos \phi_1},$$

$$\tan \theta_u = -\frac{\lambda \sqrt{\mu_1^{u2} \cos^2 \phi_1 + \mu_2^{u2} \sin^2(\phi_1 - \phi_2)}}{\sqrt{(\cos^2 \phi_2 + \lambda^2 \cos^2 \phi_1) \mu_1^{u2} + \lambda^2 \sin^2(\phi_1 - \phi_2) \mu_2^{u2}} + \mu_1^u \cos \phi_2}.$$

The CKM matrix is also written analytically using the expressions Eqs. (14) and (16) as

$$\begin{aligned}
V_{\text{CKM}} &= V_u^\dagger V_d = S_u U_u^T U_d^* S_d^\dagger \\
&\approx S_u \begin{pmatrix} \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d e^{i(-\theta_3^u + \theta_3^d)} & \sin \theta_u \cos \theta_d e^{i\theta_3^u} - \cos \theta_u \sin \theta_d e^{i\theta_3^d} & (\lambda \cos \theta_d e^{i\phi_1} - \sin \theta_d e^{i\phi_2 + i\theta_3^d}) \frac{\mu_3^u}{\mu_0^u} - (\lambda \cos \theta_d e^{-i\phi_1} - \sin \theta_d e^{-i\phi_2 + i\theta_3^d}) \frac{\mu_3^d}{\mu_0^d} \\ \cos \theta_u \sin \theta_d e^{-i\theta_3^d} - \sin \theta_u \cos \theta_d e^{-i\theta_3^u} & \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d e^{i\theta_3^u - i\theta_3^d} & (\lambda \sin \theta_d e^{i\phi_1 - i\theta_3^d} + \cos \theta_d e^{i\phi_2}) \frac{\mu_3^u}{\mu_0^u} - (\lambda \sin \theta_d e^{-i\phi_1 - i\theta_3^d} + \cos \theta_d e^{-i\phi_2}) \frac{\mu_3^d}{\mu_0^d} \\ (\lambda \cos \theta_u e^{i\phi_1} - \sin \theta_u e^{i\phi_2 - i\theta_3^u}) \frac{\mu_3^d}{\mu_0^d} - (\lambda \cos \theta_u e^{-i\phi_1} - \sin \theta_u e^{-i\phi_2 - i\theta_3^u}) \frac{\mu_3^u}{\mu_0^u} & (\lambda \sin \theta_u e^{i\phi_1 + i\theta_3^u} + \cos \theta_u e^{i\phi_2}) \frac{\mu_3^d}{\mu_0^d} - (\lambda \sin \theta_u e^{-i\phi_1 + i\theta_3^u} + \cos \theta_u e^{-i\phi_2}) \frac{\mu_3^u}{\mu_0^u} \end{pmatrix} S_d^\dagger.
\end{aligned} \tag{17}$$

Next, we examine our model numerically. The present experimental data for quark masses and CKM matrix are given in the PDG 2008 [27];

$$\begin{aligned}
\frac{m_d}{m_s} &= 0.045 \pm 0.025, \quad \frac{m_s}{m_b} = 0.025 \pm 0.008, \quad m_b = 4.20 \pm 0.12 \text{ GeV}, \\
\frac{m_u}{m_c} &= 0.0019 \pm 0.0008, \quad \frac{m_c}{m_t} = 0.0074 \pm 0.0006, \quad m_t = 171.3 \pm 2.3 \text{ GeV}, \\
|V_{\text{CKM}}| &= \begin{pmatrix} 0.9740 \text{ to } 0.9744 & 0.2247 \text{ to } 0.2267 & 0.0034 \text{ to } 0.0038 \\ 0.2246 \text{ to } 0.2266 & 0.9731 \text{ to } 0.9736 & 0.0404 \text{ to } 0.0425 \\ 0.0084 \text{ to } 0.0090 & 0.0397 \text{ to } 0.0417 & 0.9991 \text{ to } 0.9992 \end{pmatrix}, \tag{18} \\
\text{vertex coordinate of unitarity triangle } \bar{\rho} &= 0.135_{-0.016}^{+0.031}, \quad \bar{\eta} = 0.349_{-0.017}^{+0.015}, \\
\text{invariant measure of CP violation } J &= (3.05_{-0.20}^{+0.19}) \times 10^{-5},
\end{aligned}$$

where vertex coordinate of unitarity triangle ($\bar{\rho}$, $\bar{\eta}$) and Jarlskog invariant measure of CP violation J are defined as

$$\begin{aligned}
\bar{\rho} &= \text{Re} \left[\frac{(V_{\text{CKM}})_{11} (V_{\text{CKM}}^*)_{13}}{(V_{\text{CKM}})_{21} (V_{\text{CKM}}^*)_{23}} \right], \quad \bar{\eta} = \text{Im} \left[\frac{(V_{\text{CKM}})_{11} (V_{\text{CKM}}^*)_{13}}{(V_{\text{CKM}})_{21} (V_{\text{CKM}}^*)_{23}} \right], \\
J &= \text{Im} [(V_{\text{CKM}})_{12} (V_{\text{CKM}})_{23} (V_{\text{CKM}}^*)_{13} (V_{\text{CKM}}^*)_{22}]. \tag{18'}
\end{aligned}$$

Using the computer simulation, we estimate the allowed region for values of 11 parameters (μ_0^d , μ_1^d , μ_2^d , μ_3^d , μ_0^u , μ_1^u , μ_2^u , μ_3^u , λ , ϕ_1 , ϕ_2) so that the quark masses and V_{CKM} computed

from these parameters satisfy the experimental data Eq. (18). The estimated results are following values;

$$\begin{aligned}
\mu_0^d &= 4.20 \pm 0.12 \text{ GeV}, \quad \frac{\mu_1^d}{\mu_0^d} = 0.0120 \pm 0.0030, \quad \frac{\mu_2^d}{\mu_0^d} = -0.0136 \pm 0.0004, \\
\frac{\mu_3^d}{\mu_0^d} &= \pm(0.0282 \pm 0.0008), \\
\mu_0^u &= 171.3 \pm 2.3 \text{ GeV}, \quad \frac{\mu_1^u}{\mu_0^u} = 0.00369 \pm 0.00003, \quad \frac{\mu_2^u}{\mu_0^u} = -0.00378 \pm 0.00003, \\
\frac{\mu_3^u}{\mu_0^u} &= \mp(0.0127 \pm 0.0007) \quad (\text{opposite sign for that of the ratio } \mu_3^d/\mu_0^d), \\
\lambda &= 0.207 \pm 0.004, \quad \phi_1 = -(74.9 \pm 0.8)^\circ, \quad \phi_2 = (0.74 \pm 0.31)^\circ.
\end{aligned} \tag{19}$$

For these values of parameters, three mixing angles and a CP-phase in standard expression Eq. (12) for V_{CKM} are estimated as follows

$$\theta_{12} = (13.1 \pm 0.1)^\circ, \quad \theta_{23} = (2.38 \pm 0.05)^\circ, \quad \theta_{13} = (0.207 \pm 0.009)^\circ, \quad \delta = (68.1 \pm 3.8)^\circ. \tag{20}$$

Finally, we consider what are the origin of the Cabibbo angle and CP violation phase in our model. $|V_{\text{CKM}}|_{12}$ elements is expressed approximately in Eq. (17) as $|\cos \theta_u \sin \theta_d e^{-i\theta_3^d} - \sin \theta_u \cos \theta_d e^{-i\theta_3^u}|$. This is expressed by using the approximation $\theta_3^d \approx -\theta_3^u \approx \phi_1$ and $\theta_d \approx \theta_u$, which are recognized from the expression for θ_3^d, θ_3^u in Eqs. (14) and (16) and the values Eq. (19), as follows

$$\begin{aligned}
|\cos \theta_u \sin \theta_d e^{-i\phi_1} - \sin \theta_u \cos \theta_d e^{i\phi_1}| &= [(\cos \theta_u \sin \theta_d - \sin \theta_u \cos \theta_d)^2 \cos^2 \phi_1 + \\
&(\cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d)^2 \sin^2 \phi_1]^{1/2} \approx |\sin(\theta_d + \theta_u) \sin \phi_1| \approx |\lambda \sin \phi_1|.
\end{aligned}$$

We can say that the origin of Cabibbo angle and CP violation phase are the ratio $\lambda = |H_1^0/H_2^0|$ and the relative phase ϕ_1 between phase of H_1^0 and H_S^0 .

IV. LEPTON MASS AND MIXING

In this section, we consider the charged lepton and neutrino mass hierarchy and neutrino mixing V_{MNS} , which has the tri-bimaximal-like mixing character. We assume the Dirac mass terms Eq. (6) for mass of charged lepton (e, μ, τ) and Dirac mass of (ν_e, ν_μ, ν_τ) and the Majorana mass term Eq. (8) for Majorana mass. For charged leptons (e, μ, τ), mass term

is represented in Eq. (6) as

$$-\mathcal{L}_D^l = \overline{l^L} M_l l^R + h.c.,$$

$$l^{L,R} = \begin{pmatrix} e_{L,R} \\ \mu_{L,R} \\ \tau_{L,R} \end{pmatrix}, \quad M_l = \begin{pmatrix} \mu_1^l + \mu_2^l e^{i\phi_2} & \lambda \mu_2^l e^{i\phi_1} & \lambda \mu_3^l e^{i\phi_1} \\ \lambda \mu_2^l e^{i\phi_1} & \mu_1^l - \mu_2^l e^{i\phi_2} & \mu_3^l e^{i\phi_2} \\ \lambda \mu_3^l e^{i\phi_1} & \mu_3^l e^{i\phi_2} & \mu_0^l \end{pmatrix}.$$

This mass matrix M_l is diagonalized by the formula similar to Eq. (9),

$$V_l^\dagger M_l U_l = \text{diag}[m_e, m_\mu, m_\tau], \quad V_l = U_l^* S_l^\dagger, \quad S_l = \text{diag}[e^{i\alpha_l}, e^{i\beta_l}, e^{i\gamma_l}],$$

$\alpha_l, \beta_l, \gamma_l$ are arbitrary real constants,

$$(e_R^m, \mu_R^m, \tau_R^m)^T = U_l^\dagger (e_R, \mu_R, \tau_R)^T, \quad (e_L^m, \mu_L^m, \tau_L^m)^T = V_l^\dagger (e_L, \mu_L, \tau_L)^T, \quad (21)$$

where $(m_e^2, m_\mu^2, m_\tau^2)$ are eigenvalues of $M_l M_l^\dagger$ and (e^m, μ^m, τ^m) are the mass eigen states.

For the neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$, we assume the Dirac mass Eq. (6) and Majorana mass Eq. (8),

$$-\mathcal{L}_{D+M}^\nu = \overline{\nu^L} M_D^\nu \nu^R - \frac{1}{2} (\nu^R)^T C^{-1} M_M \nu^R + h.c., \quad (22)$$

where

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad M_D^\nu = \begin{pmatrix} \mu_1^\nu + \mu_2^\nu e^{-i\phi_2} & \lambda \mu_2^\nu e^{-i\phi_1} & \lambda \mu_3^\nu e^{-i\phi_1} \\ \lambda \mu_2^\nu e^{-i\phi_1} & \mu_1^\nu - \mu_2^\nu e^{-i\phi_2} & \mu_3^\nu e^{-i\phi_2} \\ \lambda \mu_3^\nu e^{-i\phi_1} & \mu_3^\nu e^{-i\phi_2} & \mu_0^\nu \end{pmatrix},$$

$$M_M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_0 \end{pmatrix}.$$

Using the relation,

$$\overline{\nu^L} M_D^\nu \nu^R = \frac{1}{2} (\overline{\nu^L} M_D^\nu \nu^R + \overline{\hat{\nu}^L} (M_D^\nu)^T \hat{\nu}^R),$$

where $\hat{\nu}$ is an anti-neutrino of ν , and $\hat{\nu}^R = C \overline{\nu^L}^T$, $\overline{\hat{\nu}^L} = -\nu^{R^T} C^{-1}$, we can rewrite the neutrino mass terms Eq. (22) to the 6×6 matrix as

$$-\mathcal{L}_{D+M}^\nu = \frac{1}{2} \left[(\overline{\nu^L}, \overline{\hat{\nu}^L}) \begin{pmatrix} 0 & M_D^\nu \\ M_D^{\nu^T} & M_M \end{pmatrix} \begin{pmatrix} \hat{\nu}^R \\ \nu^R \end{pmatrix} + h.c. \right]. \quad (23)$$

Because $M_D^\nu \ll M_M$, this 6×6 mass matrix in Eq. (23) is diagonalized by the 3×3 matrices as

$$-\mathcal{L}_{D+M}^\nu \approx \frac{1}{2} \left[\begin{pmatrix} \overline{\nu^L}, & \overline{\hat{\nu}^L} \end{pmatrix} \begin{pmatrix} -M_D^\nu M_M^{-1} M_D^{\nu T} & 0 \\ 0 & M_M \end{pmatrix} \begin{pmatrix} \hat{\nu}^R \\ \nu^R \end{pmatrix} + \right. \\ \left. \begin{pmatrix} \overline{\hat{\nu}^R}, & \overline{\nu^R} \end{pmatrix} \begin{pmatrix} -(M_D^\nu M_M^{-1} M_D^{\nu T})^\dagger & 0 \\ 0 & (M_M)^\dagger \end{pmatrix} \begin{pmatrix} \nu^L \\ \hat{\nu}^L \end{pmatrix} \right].$$

The (1,1) block of the matrix in this equation, $M_\nu^M = -M_D^\nu M_M^{-1} M_D^{\nu T}$ is very small compared to the (2,2) block, M_M . This result is the see-saw mechanism. We analyze the mass terms responsible for the mass matrix $M_\nu^M = -M_D^\nu M_M^{-1} M_D^{\nu T}$.

$$-\mathcal{L}_{D+M}^\nu \approx \frac{1}{2} \left[\overline{\nu^L} M_\nu^M \hat{\nu}^R + \overline{\hat{\nu}^R} M_\nu^{M\dagger} \nu^L \right], \quad (24)$$

$$M_\nu^M = -M_D^\nu M_M^{-1} M_D^{\nu T}.$$

M_D^ν is a complex symmetric matrix and M_M is a real symmetric matrix, then M_ν^M is a complex symmetric matrix. Then the M_ν^M can be diagonalized by the unitary matrix U_ν and V_ν , as for charged leptons,

$$V_\nu^\dagger M_\nu^M U_\nu = \text{diag}[m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}], \quad V_\nu = U_\nu^*, \quad (25)$$

$$(\widehat{\nu_e^m}^R, \widehat{\nu_\mu^m}^R, \widehat{\nu_\tau^m}^R)^T = U_\nu^\dagger (\widehat{\nu_e^R}, \widehat{\nu_\mu^R}, \widehat{\nu_\tau^R})^T, \quad (\nu_e^m, \nu_\mu^m, \nu_\tau^m)^T = V_\nu^\dagger (\nu_e^L, \nu_\mu^L, \nu_\tau^L)^T,$$

where $(m_{\nu_e}^2, m_{\nu_\mu}^2, m_{\nu_\tau}^2)$ are eigenvalues of $M_\nu^M M_\nu^{M\dagger}$ and $\nu^m = (\nu_e^m, \nu_\mu^m, \nu_\tau^m)^T$ are the mass eigen states. Here, it should be pointed out that the relation $V_\nu = U_\nu^*$ in present neutrino case is caused from the fact that the Majorana mass term $\overline{\nu^L} M_\nu^M \hat{\nu}^R = -\hat{\nu}^{R T} C^{-1} M_\nu^M \hat{\nu}^R$ is consisted from only the $\hat{\nu}^R$ field. Using the mass eigen states ν^m , we rewrite Eq. (24) to

$$-\mathcal{L}_{D+M}^\nu \approx \frac{1}{2} \left[\overline{\nu^m L} \text{diag}[m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}] \widehat{\nu^m}^R + \overline{\widehat{\nu^m}^R} \text{diag}[m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}] \nu^m L \right] \\ = \frac{1}{2} \overline{\chi^m} \text{diag}[m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}] \chi^m, \quad \chi^m = \nu^m L + \widehat{\nu^m}^R. \quad (26)$$

where χ^m is a Majorana neutrino. The mixing matrix of neutrinos corresponding to the V_{CKM} in quark sector, V_{MNS} is expressed as

$$V_{\text{MNS}} = V_l^\dagger V_\nu = S_l U_l^T U_\nu^*. \quad (27)$$

This V_{MNS} is parametrized by three mixing angles and one Dirac CP-violation phase and two Majorana CP-violation phases, adjusting 3 arbitrary phase parameters in S_l . Standard

expression of this matrix is written as

$$V_{\text{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_M, \quad (28)$$

where $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$, δ is CP-violating Dirac phase,

$P_M = \text{diag}[1, e^{i\frac{\beta}{2}}, e^{i\frac{\gamma}{2}}]$, β, γ are CP-violating Majorana phases.

Diagonalizing M_ν^M , we calculate the eigenvalues of Majorana neutrino mass and neutrino mixing matrix. First, we calculate $M_\nu^M = -M_D^\nu M_M^{-1} M_D^{\nu T}$, neglecting λ^2 and ϕ_2 because of the results that $\lambda \approx 0.21$, $\phi_2 \approx 0$ obtained in the quark sector numerical analysis,

$$M_\nu^M = -M_D^\nu M_M^{-1} M_D^{\nu T} \\ \approx - \begin{pmatrix} \frac{(\mu_1^\nu + \mu_2^\nu)^2}{M_1} & \lambda \left(\frac{2\mu_1^\nu \mu_2^\nu}{M_1} + \frac{\mu_3^{\nu 2}}{M_0} \right) e^{-i\phi_1} & \lambda \left(\frac{\mu_3^\nu (\mu_1^\nu + 2\mu_2^\nu)}{M_1} + \frac{\mu_0^\nu \mu_3^\nu}{M_0} \right) e^{-i\phi_1} \\ \lambda \left(\frac{2\mu_1^\nu \mu_2^\nu}{M_1} + \frac{\mu_3^{\nu 2}}{M_0} \right) e^{-i\phi_1} & \frac{(\mu_1^\nu - \mu_2^\nu)^2}{M_1} + \frac{\mu_3^{\nu 2}}{M_0} & \frac{\mu_3^\nu (\mu_1^\nu - \mu_2^\nu)}{M_1} + \frac{\mu_0^\nu \mu_3^\nu}{M_0} \\ \lambda \left(\frac{\mu_3^\nu (\mu_1^\nu + 2\mu_2^\nu)}{M_1} + \frac{\mu_0^\nu \mu_3^\nu}{M_0} \right) e^{-i\phi_1} & \frac{\mu_3^\nu (\mu_1^\nu - \mu_2^\nu)}{M_1} + \frac{\mu_0^\nu \mu_3^\nu}{M_0} & \frac{\mu_3^{\nu 2}}{M_1} + \frac{\mu_0^{\nu 2}}{M_0} \end{pmatrix}.$$

Using the assumption $M_1 \ll M_0$ and $|\mu_1^\nu| \approx |\mu_2^\nu| \approx |\mu_3^\nu| \ll |\mu_0^\nu|$, M_ν^M can be expressed as

$$M_\nu^M \approx -\frac{1}{M_1} \begin{pmatrix} (\mu_1^\nu + \mu_2^\nu)^2 & 2\lambda \mu_1^\nu \mu_2^\nu e^{-i\phi_1} & \\ 2\lambda \mu_1^\nu \mu_2^\nu e^{-i\phi_1} & (\mu_1^\nu - \mu_2^\nu)^2 & \\ \lambda \left(\mu_3^\nu (\mu_1^\nu + 2\mu_2^\nu) + \frac{M_1 \mu_0^\nu \mu_3^\nu}{M_0} \right) e^{-i\phi_1} & \mu_3^\nu (\mu_1^\nu - \mu_2^\nu) + \frac{M_1 \mu_0^\nu \mu_3^\nu}{M_0} & \\ & \lambda \left(\mu_3^\nu (\mu_1^\nu + 2\mu_2^\nu) + \frac{M_1 \mu_0^\nu \mu_3^\nu}{M_0} \right) e^{-i\phi_1} & \\ & \mu_3^\nu (\mu_1^\nu - \mu_2^\nu) + \frac{M_1 \mu_0^\nu \mu_3^\nu}{M_0} & \\ & \mu_3^{\nu 2} + \frac{M_1 \mu_0^{\nu 2}}{M_0} & \end{pmatrix}. \quad (29)$$

It is recognized that this mass matrix M_ν^M has a character that ν_μ - ν_τ mixing becomes maximal and ν_e - ν_μ mixing can be large. This can be confirmed from an approximation to neglect the terms including M_1/M_0 , and to set $\mu_1^\nu - \mu_2^\nu \approx |\mu_3^\nu|$, $\mu_1^\nu + \mu_2^\nu = \delta \ll \mu_3^\nu$ induced from the assumption $\mu_1^\nu \approx -\mu_2^\nu \approx |\mu_3^\nu| \ll \mu_0$, which is obtained in previous quark sector analysis. Thus the mass matrix Eq. (29) can be parametrized as

$$M_\nu^M \approx -\frac{1}{M_1} \begin{pmatrix} \delta^2 & -\frac{\lambda}{2} \mu_3^{\nu 2} e^{-i\phi_1} & -\frac{\lambda}{2} \mu_3^\nu (\mu_3^\nu - 3\delta) e^{-i\phi_1} \\ -\frac{\lambda}{2} \mu_3^{\nu 2} e^{-i\phi_1} & \mu_3^{\nu 2} & \mu_3^{\nu 2} \\ -\frac{\lambda}{2} \mu_3^\nu (\mu_3^\nu - 3\delta) e^{-i\phi_1} & \mu_3^{\nu 2} & \mu_3^{\nu 2} \end{pmatrix}. \quad (30)$$

This mass matrix can be diagonalized analytically by the unitary matrix U_ν and U_ν^T as given in Eq. (25),

$$\begin{aligned}
U_\nu^T M_\nu^M U_\nu &\approx \text{diag} \left(\frac{\delta^2}{2M_1}(-\sqrt{1+a^2}+1), \frac{\delta^2}{2M_1}(\sqrt{1+a^2}+1), \frac{2\mu_3^2}{M_1} \right), \\
\text{where } a &= \frac{3\lambda\mu_3}{\sqrt{2}\delta}. \\
U_\nu &\approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & \eta e^{i\phi_1} \\ 0 & 1 & 0 \\ -\eta^{-i\phi_1} & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi_1} & 0 \\ -\sin\theta e^{i\phi_1} & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} S_\nu, \quad (31) \\
\text{where } \eta &\approx -\frac{\lambda}{2\sqrt{2}}(1 - \frac{3\delta}{2\mu_3}), \quad \tan 2\theta \approx \frac{3\lambda\mu_3}{\sqrt{2}\delta}, \\
S_\nu &= \begin{pmatrix} e^{i\frac{\pi}{2}} & 0 & 0 \\ 0 & e^{i\phi_1+i\frac{\pi}{2}} & 0 \\ 0 & 0 & e^{i\frac{\pi}{2}} \end{pmatrix}.
\end{aligned}$$

From this approximate expression, it is recognized that ν_μ - ν_τ mixing becomes maximal and ν_e - ν_μ mixing angle can be large, for example, if $\delta \approx \lambda\mu_3$, $\tan 2\theta \approx \frac{3}{\sqrt{2}}$. Furthermore, it is recognized that $|V_{\text{MNS}}|_{13} \approx \eta \approx \lambda/2\sqrt{2}$ is small but not 0. Thus, the tri-bimaximal-like mixing character of neutrino can be explained dynamically without any other symmetry than S_3 symmetry.

Next, we examine our model numerically. For charged lepton sector, experimental data of the masses of (e, μ, τ) are given in PDG 2008 [27] as,

$$\frac{m_e}{m_\mu} = 0.004836 \pm 0.000001, \quad \frac{m_\mu}{m_\tau} = 0.05946 \pm 0.00001, \quad m_\tau = 1776.84 \pm 0.17 \text{ MeV}. \quad (32)$$

For the neutrino, the experimental data obtained from the neutrino oscillation is summarized in Ref. [12], as

$$\begin{aligned}
6.90 \times 10^{-5} \text{ eV}^2 &< \Delta m_\odot^2 < 8.20 \times 10^{-5} \text{ eV}^2, \\
0.27 &< \sin^2 \theta_\odot < 0.37, \\
2.15 \times 10^{-3} \text{ eV}^2 &< |\Delta m_{\text{atm}}^2| < 2.90 \times 10^{-3} \text{ eV}^2, \\
0.33 &< \sin^2 \theta_{\text{atm}} < 0.65, \\
[V_{\text{MNS}}]_{13}^2 &< 0.052,
\end{aligned} \quad (33)$$

where these values are including 3σ uncertainty. We assume that $|\Delta m_{\text{atm}}^2| = |\Delta m_{31}^2| \cong |\Delta m_{32}^2|$, $\Delta m_\odot^2 = \Delta m_{21}^2$, where $\Delta m_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$, and $\theta_{\text{atm}} = \theta_{23}$ mixing angle, $\theta_\odot = \theta_{12}$

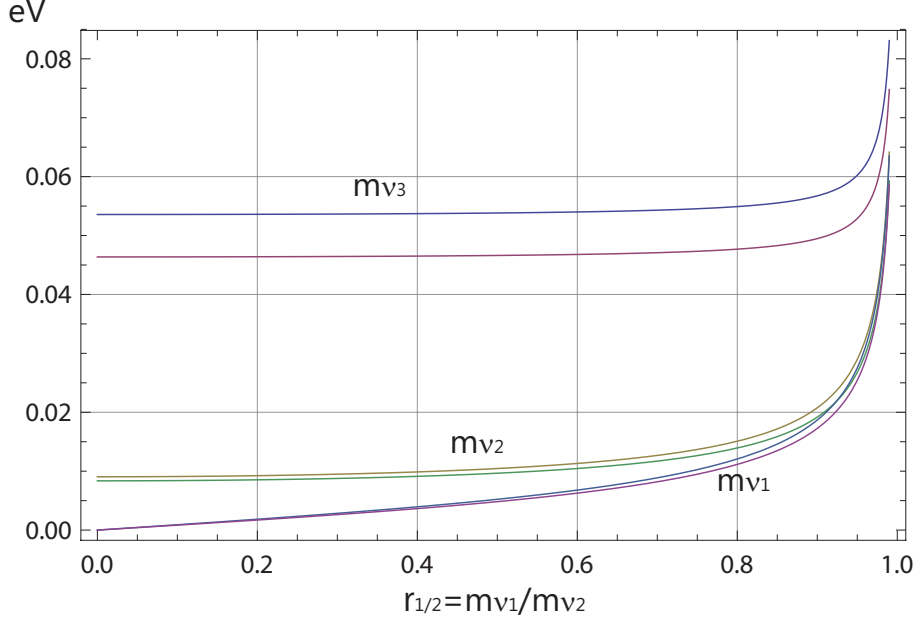


FIG. 1: Values of m_{ν_3} , m_{ν_2} , m_{ν_1} as a function of ratio $r_{1/2} = m_{\nu_1}/m_{\nu_2}$. This figure is obtained from the values of $\Delta m_{\odot}^2 = \Delta m_{21}^2$ and $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2$ in Ref. [12], assuming the normal hierarchy $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$. Upper two lines denote the m_{ν_3} values with 3σ error, middle two lines the m_{ν_2} values and the lower lines the m_{ν_1} values with 3σ error.

mixing angle. In our analysis, we assume a normal hierarchical (NM) mass spectrum $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$ for the neutrino masses. From the experimental data for mass squared differences Δm_{atm}^2 , Δm_{\odot}^2 , we can obtain the numerical values of m_{ν_1} , m_{ν_2} , m_{ν_3} as a function of ratio $r_{1/2} = m_{\nu_1}/m_{\nu_2}$; $m_{\nu_2} = \sqrt{\frac{\Delta m_{21}^2}{1 - r_{1/2}^2}}$, $m_{\nu_1} = r_{1/2} m_{\nu_2}$, $m_{\nu_3} = \sqrt{\Delta m_{31}^2 + m_{\nu_1}^2}$, as shown in Fig. 1. For the value near 1 of the parameter $r_{1/2}$, the quasi-degenerate (QD) mass spectrum $m_{\nu_1} \cong m_{\nu_2} \cong m_{\nu_3}$ appears. From the experimental data for mixing angles, the magnitudes of elements of V_{MNS} are restricted as

$$|V_{\text{MNS}}| = \begin{pmatrix} 0.77 \sim 0.88 & 0.46 \sim 0.61 & 0.00 \sim 0.26 \\ 0.10 \sim 0.49 & 0.47 \sim 0.78 & 0.57 \sim 0.81 \\ 0.28 \sim 0.61 & 0.34 \sim 0.71 & 0.57 \sim 0.81 \end{pmatrix}. \quad (34)$$

Using Eqs. (6), (21), (22), (24), (25), (27) and the numerical result (19) for λ , ϕ_1 and ϕ_2 determined in quark sector analysis, (32) for charged lepton mass and experimental data

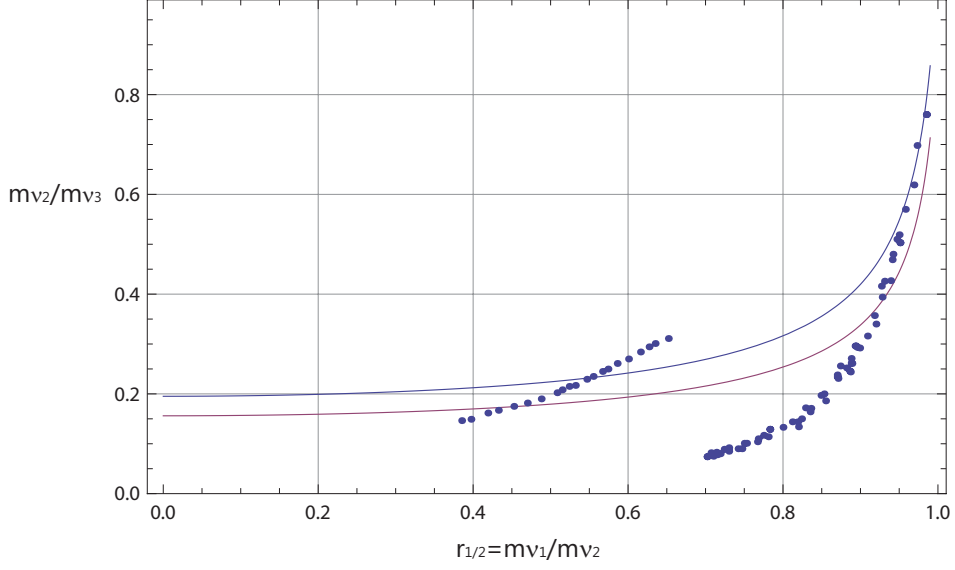


FIG. 2: The ratio m_{ν_2}/m_{ν_3} as a function of the ratio $r_{1/2} = m_{\nu_1}/m_{\nu_2}$. Two lines are obtained from the experimental data (Ref. [12]). Sequence of dots plotted in middle is obtained by satisfying the allowed region (34) of neutrino mixing $|V_{\text{MNS}}|$ in the case of $\phi_1 = 0^\circ$, $\phi_2 = 0^\circ$, where $\phi_{1,2}$ are included in mass matrices, Eq. (6). Sequence of dots plotted in right-hand side corresponds to the case of $\phi_1 = -74.9^\circ$, $\phi_2 = 0.74^\circ$, which are determined in quark sector analysis.

(34) for neutrino mixing, we estimate the allowed values for parameters μ_i^l and M_i ;

$$\lambda = 0.207 \pm 0.004, \quad \phi_1 = -(74.9 \pm 0.8)^\circ, \quad \phi_2 = (0.74 \pm 0.31)^\circ$$

(in-put data determined from quark sector analysis),

$$\mu_0^l = 1776.84 \pm 0.17 \text{ MeV}, \quad \frac{\mu_1^l}{\mu_0^l} = 0.0308 \pm 0.0007, \quad \frac{\mu_2^l}{\mu_0^l} = -(0.0307 \pm 0.0017), \quad (35)$$

$$\frac{\mu_3^l}{\mu_0^l} = -0.0233 \sim 0.0233, \quad \frac{M_1}{M_0} = 0.0016 \pm 0.0004.$$

Values of ratio m_{ν_1}/m_{ν_2} and m_{ν_2}/m_{ν_3} calculated from the values of μ_i^l satisfying the allowed values (Eq. (34)) of neutrino mixing $|V_{\text{MNS}}|$ are plotted by sequences of dots in FIG. 2. In FIG. 2, sequence of dots in middle is obtained in the case $\phi_1 = 0^\circ$, $\phi_2 = 0^\circ$, sequence of dots in right-hand side corresponds to the case $\phi_1 = -74.9^\circ$, $\phi_2 = 0.74^\circ$, which are determined in quark sector analysis. From this figure, it is shown that the allowed values of ratios m_{ν_1}/m_{ν_2} and m_{ν_2}/m_{ν_3} are restricted, and for these restricted ratios, the value of $|V_{\text{MNS}}|_{13}$ is

determined as follows,

$$\begin{aligned}
\frac{m_{\nu_1}}{m_{\nu_2}} &= 0.44 \sim 0.56, \quad \frac{m_{\nu_2}}{m_{\nu_3}} = 0.17 \sim 0.23, \quad |V_{\text{MNS}}|_{13} = 0.042 \sim 0.065, \\
&\text{for } \phi_1 = 0^\circ, \phi_2 = 0^\circ \text{ case,} \\
\frac{m_{\nu_1}}{m_{\nu_2}} &= 0.92 \sim 0.98, \quad \frac{m_{\nu_2}}{m_{\nu_3}} = 0.35 \sim 0.75, \quad |V_{\text{MNS}}|_{13} = 0.056 \sim 0.080, \\
&\text{for } \phi_1 = -74.9^\circ, \phi_2 = 0.74^\circ \text{ case.}
\end{aligned} \tag{36}$$

Although the case of $\phi_1 = -74.9^\circ$, $\phi_2 = 0.74^\circ$ is interested in present analysis, but the case of $\phi_1 = 0.0^\circ$, $\phi_2 = 0.0^\circ$ is presented for a comparison. In previous our analysis [19], we studied the case of $\phi_1 = 0.0^\circ$, $\phi_2 = 0.0^\circ$, and obtained the result, $m_{\nu_1}/m_{\nu_2} = 0.36 \sim 0.49$, $|V_{\text{MNS}}|_{13} = 0.04 \sim 0.06$. For the case of $\phi_1 = -74.9^\circ$, $\phi_2 = 0.74^\circ$, it is predicted that the neutrino spectroscopy is near the quasi-degenerate(QD) spectrum, and $|V_{\text{MNS}}|_{13}$ is not so tiny.

The allowed values of parameters μ_1^ν , μ_2^ν , μ_3^ν are determined from the allowed regions of m_{ν_1}/m_{ν_2} , m_{ν_2}/m_{ν_3} in Eq. (36), as

$$\begin{aligned}
m_{\nu_3} &\approx \frac{2\mu_3^{\nu 2}}{M_1} = (0.047 \sim 0.053)\text{eV}, \quad \frac{\mu_1^\nu}{\mu_0^\nu} = 0.045 \sim 0.050, \quad \frac{\mu_2^\nu}{\mu_0^\nu} = -0.014 \sim -0.019, \\
\frac{\mu_3^\nu}{\mu_0^\nu} &= \pm(0.034 \sim 0.047), \quad \text{for the case } \phi_1 = 0.0^\circ, \phi_2 = 0.0^\circ, \\
m_{\nu_3} &\approx \frac{2\mu_3^{\nu 2}}{M_1} = (0.05 \sim 0.07)\text{eV}, \quad \frac{\mu_1^\nu}{\mu_0^\nu} = 0.035 \sim 0.038, \quad \frac{\mu_2^\nu}{\mu_0^\nu} = -0.001 \sim -0.007, \\
\frac{\mu_3^\nu}{\mu_0^\nu} &= \pm(0.005 \sim 0.023), \quad \text{for the case } \phi_1 = -74.9^\circ, \phi_2 = 0.74^\circ.
\end{aligned} \tag{37}$$

From the result $\frac{2\mu_3^{\nu 2}}{M_1} \approx 0.06\text{eV}$, $\frac{\mu_3^\nu}{\mu_0^\nu} \approx 0.03$ in Eq. (37) and $\frac{M_1}{M_0} = 0.0016$ in Eq. (35), and assumption $\mu_0^\nu \sim \mu_0^l \frac{\mu_0^u}{\mu_0^d} \approx 73.3\text{GeV}$ in Eq.(19), M_1 , M_0 are estimated as

$$M_1 \approx 1.6 \times 10^{11}\text{GeV}, \quad M_0 \approx 10^{14}\text{GeV}, \tag{38}$$

which is compatible with the result in GUT, $M_{\text{GUT}} \approx 2 \times 10^{16}\text{GeV}$. For the case $\phi_1 = -74.9^\circ$, $\phi_2 = 0.74^\circ$, Dirac CP-violation phase δ is produced and this phase generates CP violation in neutrino oscillation. The magnitude of CP violation in $\nu_l \rightarrow \nu_{l'}$ and $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ oscillations is determined by the invariant measure of CP violation, J , as the same definition as in Eq. (18'),

$$J = \text{Im} [(V_{\text{MNS}})_{12}(V_{\text{MNS}})_{23}(V_{\text{MNS}}^*)_{13}(V_{\text{MNS}}^*)_{22}],$$

where $(V_{\text{MNS}})_{ij}$ is a (i, j) element of matrix in Eq. (28) without the Majorana phase matrix P_M , because Majorana phases in P_M do not contribute in neutrino oscillation [28]. In our model, neutrinos are assumed to be the Majorana fermions produced through the see-saw mechanism. Majorana nature is found in neutrino-less double beta decay, $(\beta\beta)_{0\nu}$ -decay. The $(\beta\beta)_{0\nu}$ -decay is characterized by the effective Majorana mass $|<m>|$ defined as

$$|<m>| = |m_{\nu_1}(V_{\text{MNS}})_{11}^2 + m_{\nu_2}(V_{\text{MNS}})_{12}^2 + m_{\nu_3}(V_{\text{MNS}})_{13}^2|, \quad (39)$$

where, V_{MNS} is a matrix expressed in Eq.(28) with Majorana phase matrix P_M . We estimate the Dirac phase δ , the Majorana phases β, γ , the invariant measure J of CP violation, and effective Majorana mass $|<m>|$ for the allowed values of μ_1^ν/μ_0^ν , μ_2^ν/μ_0^ν , μ_3^ν/μ_0^ν denoted in Eq.(37),

$$\begin{aligned} \delta &= 180.0^\circ, \quad \beta = 0.0^\circ, \quad \gamma = 0.0^\circ, \quad J = 0.0, \quad |<m>| = 0.0059 \sim 0.0079, \\ &\text{for the case } \phi_1 = 0.0^\circ, \quad \phi_2 = 0.0^\circ, \\ \delta &= (65.2 \sim 84.3)^\circ, \quad \beta = (24.3 \sim 44.2)^\circ, \quad \gamma = (16.9 \sim 31.8)^\circ, \\ J &= -(0.010 \sim 0.017), \quad |<m>| = 0.026 \sim 0.048, \\ &\text{for the case } \phi_1 = -74.9^\circ, \quad \phi_2 = 0.74^\circ. \end{aligned} \quad (40)$$

Thus, for the neutrino mixing derived from the neutrino Dirac mass with $\phi_1 = -74.9^\circ, \phi_2 = 0.74^\circ$, the magnitude of CP violation in neutrino oscillation expressed by J , is predicted as to be rather larger than that of quark, which is shown in Eq. (18). Further the effective Majorana mass $|<m>|$ is predicted to be not so tiny.

V. CONCLUSION

We assumed that the weak bases of flavors (u, c) , (d, s) , (e, μ) , and Dirac neutrino (ν_e, ν_μ) are S_3 doublets and t, b, τ , and ν_τ are S_3 singlets. Further, we assumed that the Higgs S_3 doublet (H_1, H_2) and Higgs S_3 singlet H_S . From these S_3 doublets and singlets, we constructed S_3 invariant Yukawa interactions and the mass matrices for weak basis of flavors.

Obtained mass matrices for quark sector are

$$M_d = \begin{pmatrix} \mu_1^d + \mu_2^d & \lambda\mu_2^d e^{i\phi_1} & \lambda\mu_3^d e^{i\phi_1} \\ \lambda\mu_2^d e^{i\phi_1} & \mu_1^d - \mu_2^d e^{i\phi_2} & \mu_3^d e^{i\phi_2} \\ \lambda\mu_3^d e^{i\phi_1} & \mu_3^d e^{i\phi_2} & \mu_0^d \end{pmatrix}, \quad M_u = \begin{pmatrix} \mu_1^u + \mu_2^u & \lambda\mu_2^u e^{-i\phi_1} & \lambda\mu_3^u e^{-i\phi_1} \\ \lambda\mu_2^u e^{-\phi_1} & \mu_1^u - \mu_2^u e^{-\phi_2} & \mu_3^u e^{-\phi_2} \\ \lambda\mu_3^u e^{-i\phi_1} & \mu_3^u e^{-\phi_2} & \mu_0^u \end{pmatrix},$$

where $\lambda = |H_1^0|/|H_2^0|$ and $\phi_1 = \text{phase of } H_1^0 - \text{phase of } H_S^0$, and $\phi_2 = \text{phase of } H_2^0 - \text{phase of } H_S^0$. From the present experimental data for quark masses and V_{CKM} matrix including the CP violation phase [27], we can obtain the results Eq. (19),

$$\begin{aligned}\mu_0^d &= 4.20 \pm 0.12 \text{GeV}, \quad \frac{\mu_1^d}{\mu_0^d} = 0.0120 \pm 0.0030, \quad \frac{\mu_2^d}{\mu_0^d} = -(0.0136 \pm 0.0004), \\ \frac{\mu_3^d}{\mu_0^d} &= \pm(0.0282 \pm 0.0008), \\ \mu_0^u &= 171.3 \pm 2.3 \text{GeV}, \quad \frac{\mu_1^u}{\mu_0^u} = 0.00369 \pm 0.00003, \quad \frac{\mu_2^u}{\mu_0^u} = -0.00378 \pm 0.00003, \\ \frac{\mu_3^u}{\mu_0^u} &= \mp(0.0127 \pm 0.0007), \quad (\text{opposite sign for that of the ratio } \mu_3^d/\mu_0^d) \\ \lambda &= 0.207 \pm 0.004, \quad \phi_1 = -(74.9 \pm 0.8)^\circ, \quad \phi_2 = (0.74 \pm 0.31)^\circ.\end{aligned}$$

CP-phase in standard expression for V_{CKM} is

$$\delta = (68.1 \pm 3.8)^\circ.$$

In our model, the origin of the Cabibbo angle can be explained by the ratio $\lambda = |H_1^0|/|H_2^0|$ and the origin of the CP violation by the phase difference $\phi_1 = \text{phase of } H_1^0 - \text{phase of } H_S^0$.

For lepton sector, mass matrices are obtained as

$$\begin{aligned}M_l &= \begin{pmatrix} \mu_1^l + \mu_2^l e^{i\phi_2} & \lambda \mu_2^l e^{i\phi_1} & \lambda \mu_3^l e^{i\phi_1} \\ \lambda \mu_2^l e^{i\phi_1} & \mu_1^l - \mu_2^l e^{i\phi_2} & \mu_3^l e^{i\phi_2} \\ \lambda \mu_3^l e^{i\phi_1} & \mu_3^l e^{i\phi_2} & \mu_0^l \end{pmatrix}, \\ M_\nu^D &= \begin{pmatrix} \mu_1^\nu + \mu_2^\nu e^{-i\phi_2} & \lambda \mu_2^\nu e^{-i\phi_1} & \lambda \mu_3^\nu e^{-i\phi_1} \\ \lambda \mu_2^\nu e^{-i\phi_1} & \mu_1^\nu - \mu_2^\nu e^{-i\phi_2} & \mu_3^\nu e^{-i\phi_2} \\ \lambda \mu_3^\nu e^{-i\phi_1} & \mu_3^\nu e^{-i\phi_2} & \mu_0^\nu \end{pmatrix}, \quad M_M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_0 \end{pmatrix}.\end{aligned}\tag{41}$$

Diagonalizing the mass matrix M_l and $M_\nu^M = -M_D^\nu M_M^{-1} M_D^{\nu T}$ obtained through the see-saw mechanism, we obtain the charged lepton mass, neutrino mass and neutrino mixing M_{MNS} . In our model, tri-bimaximal-like mixing character of the neutrino mixing V_{MNS} is obtained dynamically from the hierarchy of mass parameters $\mu_1^\nu + \mu_2^\nu \ll \mu_1^\nu - \mu_2^\nu \approx |\mu_3^\nu| \ll \mu_0^\nu$, $M_1 \ll M_0$ and smallness of $\lambda \sim 0.21$, without any other symmetry restriction than S_3 . From the present experimental data for charged lepton mass [27] and neutrino mass and mixing

V_{MNS} [12], we obtained the allowed values for mass parameters and $|V_{\text{MNS}}|_{13}$:

$$\begin{aligned}
\lambda &= 0.207 \pm 0.004, \quad \phi_1 = -(74.9 \pm 0.8)^\circ, \quad \phi_2 = (0.74 \pm 0.31)^\circ, \\
&\quad (\text{in-put data determined from quark sector analysis,}) \\
\mu_0^l &= 1776.84 \pm 0.17 \text{MeV}, \quad \frac{\mu_1^l}{\mu_0^l} = 0.0315, \quad \frac{\mu_2^l}{\mu_0^l} = -0.0324, \quad \frac{\mu_3^l}{\mu_0^l} = -0.0233 \sim 0.0233, \\
m_{\nu_3} &\approx (0.05 \sim 0.07) \text{eV}, \quad \frac{m_{\nu_1}}{m_{\nu_2}} = 0.92 \sim 0.98, \quad \frac{m_{\nu_2}}{m_{\nu_3}} = 0.35 \sim 0.75, \\
|V_{\text{MNS}}|_{13} &= 0.056 \sim 0.080 \quad \frac{\mu_1^\nu}{\mu_0^\nu} = 0.035 \sim 0.038, \quad \frac{\mu_2^\nu}{\mu_0^\nu} = -(0.001 \sim 0.007), \\
\frac{\mu_3^\nu}{\mu_0^\nu} &= \pm(0.005 \sim 0.023), \quad M_1 \approx 1.6 \times 10^{11} \text{GeV}, \quad M_0 \approx 10^{14} \text{GeV}.
\end{aligned}$$

Thus, neutrino mass in our model favors a quasi-degenerate(QD) spectrum, and $|V_{\text{MNS}}|$ is not so tiny. We can estimate the CP violation Dirac phase δ , Majorana phases (β, γ) and CP violation measure J , effective Majorana mass $|<m>|$ in $(\beta\beta)_{0\nu}$ -decay, as

$$\begin{aligned}
\delta &= (65.2 \sim 84.3)^\circ, \quad \beta = (24.3 \sim 44.2)^\circ, \quad \gamma = (16.9 \sim 31.8)^\circ, \\
J &= -(0.010 \sim 0.017), \quad |<m>| = 0.026 \sim 0.048.
\end{aligned}$$

Appendix: Diagonalization of complex symmetric matrix

For an arbitrary complex matrix M , M can be diagonalized by the unitary matrix V and U as

$$V^\dagger M U = \text{diag}[m_1, m_2, \dots, m_n], \quad m_i \geq 0, \quad (\text{A.1})$$

where $V^\dagger M M^\dagger V = \text{diag}[m_1^2, m_2^2, \dots, m_n^2]$, and we assume that $m_i \neq m_k$ for $i \neq k$. Since M is a complex symmetric matrix, $M = M^T = U^{\dagger T} \text{diag}[m_1, m_2, \dots, m_n] V^T$, then $U^T M M^\dagger U^{\dagger T} = \text{diag}[m_1^2, m_2^2, \dots, m_n^2]$. Hence $V \text{diag}[m_1^2, m_2^2, \dots, m_n^2] V^\dagger = U^{\dagger T} \text{diag}[m_1^2, m_2^2, \dots, m_n^2] U^T$. This relation implies that

$$U^T V \text{diag}[m_1^2, m_2^2, \dots, m_n^2] = \text{diag}[m_1^2, m_2^2, \dots, m_n^2] U^T V \quad (\text{A.2})$$

Since $U^T V$ is unitary matrix, it follows from Eq. (A2) that

$$U^T V = S^\dagger, \quad S = \text{diag}[e^{i\alpha_1}, e^{i\alpha_2}, \dots, e^{i\alpha_n}], \quad (\text{A.3})$$

where α_i are arbitrary real constants. Then we can obtain $V = U^* S^\dagger$ and $S U^T M U = \text{diag}[m_1, m_2, \dots, m_n]$. Thus, we obtain a statement that if M is a complex symmetric matrix, M can be diagonalized by the unitary matrix V and U as

$$\begin{aligned} V^\dagger M U &= \text{diag}[m_1, m_2, \dots, m_n], \quad m_i \geq 0, \quad V^\dagger M M^\dagger V = \text{diag}[m_1^2, m_2^2, \dots, m_n^2], \\ V &= U^* S^\dagger, \quad S = \text{diag}[e^{i\alpha_1}, e^{i\alpha_2}, \dots, e^{i\alpha_n}], \quad \alpha_i : \text{arbitrary real constants.} \end{aligned} \quad (\text{A.4})$$

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